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Computational Mathematics

Generation of Orthogonal Polynomials in Least-Squares Approximations

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ABSTRACT: *In this paper, the least-squares approximation of a function is presented. We shall be concerned entirely with the principal of least squares as applied to functions known only at a discrete of points. The aim of this paper is to compare with the generation of least-squares approximations by using the orthogonal polynomial and powers-of-x formulations.*

Keywords: interpolation; approximation; orthogonal polynomial; smooth approximation; generating approximation

1. INTRODUCTION

Polynomial interpolation is a method of approximating the value of a function at a point by means of a polynomial passing through known functional values. The subject of least-squares approximations is concerned with a technique by which noisy functional values can be used to generate a smooth approximation to the function. This smooth approximation can be used to approximate the derivative of the function more accurately than exact approximations.

2. THE PRINCIPLE OF LEAST SQUARES

Let $f(x)$ be a function and $\{x_i\}$, $i = 1, \ldots, n$, be a sequence of data points at which we have observed values of $f(x)$ which generally will be in error. We denote $f(x_i)$, the true value at x_i, by f_i, and we denote the observed value at x_i by $\overline{f_i}$. We define $E_i = f_i - \overline{f_i}$. We shall assume that the errors at different data points are uncorrelated.

Let $\{\emptyset_j(x)\}$, j = 0, 1 , …, be a sequence of functions defined for every x_i. Then our object is to approximate $\overline{f_i}$ by a linear combination of the $\{\emptyset_j(x)\}$,

$$\overline{f}_i \approx \sum_{j=0}^{m} a_j^{(m)} \phi_j(x_i),\; i = 1, \cdots, n$$

with the $a_j^{(m)}$ to be determined so that

$$H\left(a_0^{(m)}, \cdots, a_m^{(m)}\right) \equiv \sum_{i=1}^{n} w(x_i)\left[\overline{f}_i - \sum_{j=0}^{m} a_j^{(m)} \phi_j(x_i)\right]^2$$

$$= \sum_{i=1}^{n} w(x_i) R_i^2 \qquad (1)$$

is minimized. The function w(x) is called the weight function and is assumed to be such that $w(x_i) \geq 0$, i = 1, …, n. The quantity R_i is called the residual at x_i. The superscript m on $a_j^{(m)}$ denotes the fact that the coefficient of $\emptyset_j(x)$ will generally depend on m. Having determined the $a_j^{(m)}$ so as to satisfy (1), we have then an approximation

$$y_m(x) = \sum_{j=0}^{m} a_j^{(m)} \phi_j(x) \qquad (2)$$

which is called a least-squares approximation of $f(x)$ over $\{x_i\}$. We can use this approximation not only at the points $\{x_i\}$ but also at other values of x.

If $\phi_j(x) = x^j$ and there are no more data points than parameters in the approximating polynomial, that is, $n \leq m+1$, then by making the summation in (2) the Lagrangian interpolation polynomial corresponding to the points $\{x_i\}$, we would have $y_i = \overline{f_i}$, i = 1,…,n, where $y_i = y(x_i)$. Since (1) would then be zero, this would be the desired minimum. In our case, however, we shall be concerned with the case $n > m+1$; that is, we use a number of approximating functions less than the number of data points. Thus, for example, we might approximate a function known at five data points by a polynomial of degree 1, that is, m=1. We could derive a polynomial of degree 4 passing through these five points, but such a fourth degree polynomial will not enable us to smooth the empirical data. However, as we shall see, such smoothing is possible in general when $m+1 < n$.

Graphically, this is illustrated by Figure 1. Suppose we have empirical data at five points on a function which is in fact linear as shown. The errors in the empirical data as such are much too great to allow approximating the true function by an exact linear approximation using any two points. If we pass a fourth degree polynomial through the five points, we get an approximation whose deviation from the true function is not too bad but whose derivatives are much different from those of the true function. On the other hand, the linear least-squares approximation not only lies close to the true function but also has a similar slope.

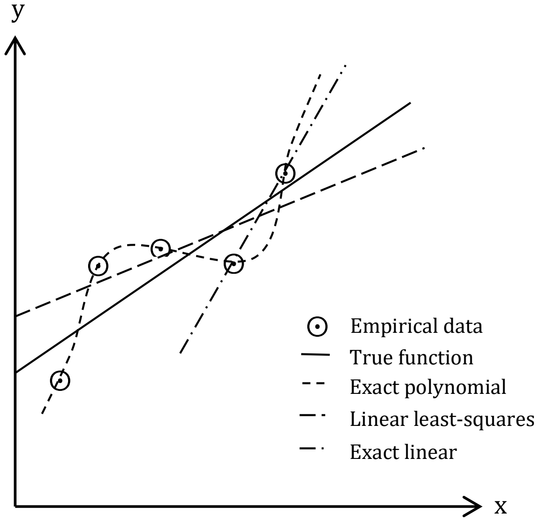

Figure 1. Least-squares and exact approximations

To calculate the $a_j^{(m)}$'s, we take the partial derivative of H in (1) with respect to $a_k^{(m)}$ and set it equal to 0, thereby obtaining

$$\frac{\partial H}{\partial a_k^{(m)}} = -2\sum_{i=1}^{n} w_i \left[\overline{f}_i - \sum_{j=0}^{m} a_j^{(m)} \phi_j(x_i) \right] \phi_k(x_i) = 0$$

$$k = 0, \cdots, m \,;\ w_i = w(x_i) \,. \qquad (3)$$

Equation (3) is a system of $m+1$ linear equations for the $m+1$ unknown $a_j^{(m)}$'s. This system is called the normal equations. If the determinant of the coefficients does not vanish, we can solve for the $a_j^{(m)}$'s. By considering

$$H\left(a_0^{(m)} + \Delta a_0, \cdots, a_m^{(m)} + \Delta a_m\right)$$

it is not hard to show that this solution is indeed a minimum.

Our basic assumption in this case is that for some unknown value of m, say M, the true function $f(x)$ can be expressed as a finite linear combination of the set of functions $\{ \phi_j(x) \}$; that is, we assume

$$f(x) = \sum_{j=0}^{M} a_j^{(M)} \phi_j(x) \,. \qquad (4)$$

3. Polynomial Least-Squares Approximations

Here we consider the case in which $\phi_j(x)$ is a polynomial of degree j. In particular, we shall consider the case $\phi_j(x) = x^j$ and $w(x) = 1$. For this case (3) becomes, after canceling the -2,

$$\sum_{i=1}^{n} \left(\overline{f}_i - \sum_{j=0}^{m} a_j^{(m)} x_i^j \right) x_i^k = 0 \,,\ k = 0, \cdots, m \,. \qquad (5)$$

Interchanging summations, we can rewrite (5) as

$$\sum_{j=0}^{m} a_j^{(m)} \left(\sum_{i=1}^{n} x_i^{j+k} \right) = \sum_{i=1}^{n} \overline{f}_i \; x_i^k \,,\ k = 0, \cdots, m \,.$$

Using the notation

$$g_{jk} = \sum_{i=1}^{n} x_i^{j+k} \,, \quad \rho_k = \sum_{i=1}^{n} \overline{f}_i \; x_i^k \,, \qquad (6)$$

the normal equations can be written

$$\sum_{j=0}^{m} g_{jk} \, a_j^{(m)} = \rho_k \,,\ k = 0, \cdots, m \,. \qquad (7)$$

Using matrix calculus, it can be proved that the least-squares problem and, thus the system (7), has a unique solution.

3.1. Solution of the Normal Equations

It seem at this point to have solved the least-squares problem for the case $\phi_j(x) = x^j$, $w(x) = 1$. All we need do is perform the perhaps tedious calculations required to solve the normal equations (7). And indeed for small values of m, say up to 5 or 6, experience indicates that the solution of (7) produces quite good least-squares approximations. But for greater values of m, the solutions found by solving (7) generally lead to progressively poorer least-squares approximations. Moreover, this is quite independent of which of the many methods available for the solution of (7) is used. An explanation of this can be found using the following argument.

For convenience let us assume that the points x_i are all in the interval (0,1). Further, let us assume that they are distributed fairly uniformly in this interval. Then g_{jk} as defined in (6) has the form of n times a Riemann sum. For large n, then, the approximation

$$g_{jk} = \sum_{i=1}^{n} x_i^{j+k} \approx n \int_0^1 x^{j+k} \, dx = \frac{n}{j+k+1} \,,$$

$$j, k = 0, \cdots, m \qquad (8)$$

should be a good one. Let $G = \left[g_{jk} \right]$ be the matrix of coefficients in (7). Using (8), we approximate G by n times the matrix H, where

$$H = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \cdots & \frac{1}{m+1} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \cdots & \frac{1}{m+2} \\ \frac{1}{3} & \cdots & \cdots & \cdots & \frac{1}{m+3} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{1}{m+1} & & & & \frac{1}{2m+1} \end{bmatrix} . \qquad (9)$$

This matrix is the principal minor of order $m+1$ of the infinite Hilbert matrix. This matrix is a classical example of an ill-conditioned matrix. A matrix is ill-conditioned if when it has been normalized so that its largest element has order of magnitude 1, as, for example, in (9), its inverse has very large elements. Thus, for example, when $m = 9$, the inverse of (9) has elements of magnitude 3×10^{12}. The result of this is that any round-off error incurred in entering the coefficients of H into the computer will result in a matrix whose true inverse has greatly magnified errors. For quite small values of m, therefore, it becomes impossible to compute an accurate solution to a set of linear equations whose coefficient matrix is H.

3.2. Choosing the Degree of the Polynomial

Our basic hypothesis is that the true function $f(x)$ is a polynomial of degree $M < n$ or at least can be accurately represented by such a polynomial. A priori we do not know what M is; our problem is to find it. If we choose a value of $m < M$, then clearly it is impossible to get a good representation of the true function. On the other hand, choosing a value of $m > M$ also defeats our purpose. We have pointed out that by choosing $m = n - 1$ we can make

$$\delta_m^2 = \sum_{i=1}^{n} w_i R_i^2 = \sum_{i=1}^{n} w_i \left(\bar{f}_i - \sum_{j=0}^{m} a_j^{(m)} x_i^j \right)^2 ,$$

$$w_i = w(x_i) \tag{10}$$

equal to 0. But in so doing, we shall have lost all smoothing properties of least-squares approximations. In fact, any value of $m > M$ sacrifices some smoothing. When we are using powers of x, (4) becomes

$$f(x) = \sum_{j=0}^{M} a_j^{(M)} x^j . \tag{11}$$

Therefore, if we knew M and calculated the least- squares approximation,

$$y_{M+1}(x) = \sum_{j=0}^{M+1} a_j^{(M+1)} x^j ,$$

using the observed data $\{ \bar{f}_i \}$, then statistically $a_{M+1}^{(M+1)}$ should be 0. That is, if there were no errors in the data, it would be 0, but because of these errors, it will not be 0 even if the assumption that $f(x)$ has the form (11) is correct. We should like then to test the statistical hypothesis that $a_{M+1}^{(M+1)} = 0$. In order to be able to do this, we make the one further assumption that the errors E_i are normally distributed with zero mean and variance $\frac{\sigma^2}{w_i}$. This assumption is reasonable because more accurate measurements, that is, those with small variance, will usually be more heavily weighted.

This statistical hypothesis that we wish to test is often called the null hypothesis. It can be tested using maximum-likelihood statistical methods; a discussion of which is beyond the scope of our paper. Here we only state the result that if the null hypothesis is correct, then the expected value of

$$\sigma_m^2 = \frac{\delta_m^2}{n - m - 1} \tag{12}$$

will be independent of m for $m = M, M+1, \cdots, n-1$. Thus in practice, since we do not know M, we would wish to solve the normal equations (7) for $m = 1, 2, \cdots$, compute σ_m^2, and continue as long as σ_m^2 decreases significantly with increasing m. As soon as a value of m is reached after which no significant decrease occurs in σ_m^2, this m is that of the null hypothesis and we have the desired least-squares approximation. In order to guard against the possibility that the underlying function is odd or even and that successive values of σ_m will therefore be nearly equal before $m = M$, in practice we should stop the computation only after several σ_m are almost the same.

4. Orthogonal-Polynomial Approximations

If $p_j(x)$ is a polynomial of degree j, the least-squares approximation of degree m can be written

$$y_m(x) = \sum_{j=0}^{m} b_j^{(m)} p_j(x) .$$

In order to minimize

$$H\left(b_0^{(m)}, \cdots, b_m^{(m)}\right) \equiv \sum_{i=1}^{n} w_i \left[\bar{f}_i - y_m(x_i) \right]^2 ,$$

we proceed as in section 2. We get, corresponding to (7),

$$\sum_{j=0}^{m} d_{jk} b_j^{(m)} = \omega_k , \ k = 0, \cdots, m , \tag{13}$$

where $d_{jk} = \sum_{i=1}^{n} w_i p_j(x_i) p_k(x_i)$,

$$\omega_k = \sum_{i=1}^{n} w_i \bar{f}_i p_k(x_i) . \tag{14}$$

For arbitrary choice of the $\{p_j(x)\}$, the computational problems involved in solving these normal equations can be just as serious as before. If, however, the $\{p_j(x)\}$ are chosen so that the nondiagonal terms of the matrix $D = \left[d_{jk} \right]$ are small compared with the diagonal elements, then the matrix D, unlike G, will not be ill-conditioned. In particular, if the $\{p_j(x)\}$ are orthogonal over the sets of points $\{x_i\}$, the off-diagonal terms will all be 0. By definition, a set of polynomials $\{p_j(x)\}$ is orthogonal over a set of points $\{x_i\}$ with respect to a weight function $w(x)$ if

$$\sum_{i=1}^{n} w_i p_j^{(n)}(x_i) p_k^{(n)}(x_i) = 0 \ \text{if} \ j \neq k , w_i = w(x_i) , \tag{15}$$

where the superscript denotes the fact that the polynomial will depend on the number of points n. We assume in what follows that $w_i > 0$ for all i. If the $\{p_j^{(n)}(x)\}$ are orthogonal, then, as defined in (14), $d_{jk} = 0$, $j \neq k$. The system (13) then becomes

$$d_{kk} b_k^{(m)} = \omega_k , \ k = 0, \cdots, m ,$$

which has the immediate solution

$$b_k^{(m)} = \frac{\omega_k}{d_{kk}} , \ k = 0, \cdots, m ,$$

thereby eliminating the problems of solving an ill-conditioned system of normal equations. Moreover, the solution with m replaced by $m + 1$ is given by

$$b_k^{(m+1)} = \frac{\omega_k}{d_{kk}} , \ k = 0, \cdots, m+1 ,$$

with ω_k and d_{kk} again given by (14). Thus

$$b_k^{(m)} = b_k^{(m+1)},\ k = 0,\cdots,m. \tag{16}$$

Therefore, to compute the solution for $m+1$, we need only compute ω_{m+1} and $d_{m+1,m+1}$. Since (16) indicates that b_k is in fact independent of m, we shall from now on drop the superscript.

We now proceed to consider the generation of polynomials orthogonal over discrete sets of points which need not be equally spaced. A convenient method for this is the Gram-Schmidt orthogonalization process. We begin with a set of m polynomials $q_j(x)$, $j = 0,1,\cdots,m-1$, where $q_j(x)$ is a polynomial of degree j, which are linearly independent over the set $\{x_i\}$. That is, there exist no constant $\{c_j\}$ other than $c_j = 0$, $j = 1,\cdots,m-1$ such that,

$$\sum_{j=0}^{m-1} c_j\, q_j(x_i) = 0,\ i = 1,\cdots,n.$$

A convenient choice for $\{q_i(x)\}$ is often $1, x, x^2,\cdots,x^{m-1}$. In any case let $p_0(x) = q_0(x)$,

$$p_j(x) = q_j(x) - \sum_{r=0}^{j-1} d_{r,j}\, p_r(x),\ j = 1,\cdots,m-1, \tag{17}$$

so that $p_j(x)$ is a polynomial of degree j. Suppose we have determined $p_0(x), p_1(x),\cdots,p_k(x)$ so that (15) is satisfied. Then, to determine $p_{k+1}(x)$ orthogonal to all $p_j(x)$, $j \le k$, we use (17) with $j = k+1$ to write

$$\sum_{i=1}^{n} w_i\, p_{k+1}(x_i)\, p_j(x_i) = \sum_{i=1}^{n} w_i\, q_{k+1}(x_i)\, p_j(x_i) - \sum_{r=0}^{k} d_{r,k+1} \sum_{i=1}^{n} w_i\, p_r(x_i)\, p_j(x_i),$$
$$j = 0,1,\cdots,k.$$

We wish the left-hand side to be 0. By orthogonality all the terms in the double summation on the right-hand side are 0 except the term in $r = j$. Therefore

$$d_{j,k+1} = \frac{\sum_{i=1}^{n} w_i\, q_{k+1}(x_i)\, p_j(x_i)}{\sum_{i=1}^{n} w_i\, p_j^2(x_i)},\quad j = 0,1,\cdots,k.$$

In this way $p_{k+1}(x)$ is determined orthogonal to all $p_j(x)$ of lower degree, and continuing this process leads to a set of m orthogonal polynomials.

A more convenient and efficient method for the derivation of orthogonal polynomials over discrete sets of points is the use of recurrence relations. Suppose that $\{p_j(x)\}$ is any sequence of polynomials satisfying the orthogonality relationship (15) with respect to some positive weight function $w(x)$ and some sequence of data points $\{x_i\}$. We shall show by induction that there exists a relation of the form

$$p_{j+1}(x) = (x - \alpha_{j+1})\, p_j(x) - \beta_j\, p_{j-1}(x),\ j = 0,1,\cdots,$$
$$p_0(x) = 1,\ p_{-1}(x) = 0, \tag{18}$$

where α_{j+1} and β_j are constants to be determined. For $j = 0$ (18) becomes

$$p_1(x) = (x - \alpha_1).$$

The relation (15) requires that

$$\sum_{i=1}^{n} w_i\, p_0(x_i)\, p_1(x_i) = \sum_{i=1}^{n} w_i\,(x_i - \alpha_1) = 0,$$

from which it follows that

$$\alpha_1 = \frac{\sum_{i=1}^{n} w_i\, x_i}{\sum_{i=1}^{n} w_i}.$$

Let us suppose that for $j = 0,1,\cdots,k$ the polynomials $p_j(x)$ satisfy a relationship of the form (18) and the orthogonality relationship (15). Then we wish to show that we can choose α_{k+1} and β_k so that with $p_{k+1}(x)$ defined by (18),

$$\sum_{i=1}^{n} w_i\, p_j(x_i)\, p_{k+1}(x_i) = 0,\ j = 0,1,\cdots,k. \tag{19}$$

Substituting (18) with $j = k$ into (19), we have

$$\sum_{i=1}^{n} w_i\, x_i\, p_j(x_i)\, p_k(x_i) - \alpha_{k+1} \sum_{i=1}^{n} w_i\, p_j(x_i)\, p_k(x_i)$$
$$-\beta_k \sum_{i=1}^{n} w_i\, p_j(x_i)\, p_{k-1}(x_i) = 0,\ j = 0,1,\cdots,k. \tag{20}$$

For $j = 0,1,\cdots,k-2$ the last two terms on the left-hand side of (20) are identically 0 by the induction hypothesis. Moreover, for these values of j, $x_i\, p_j(x_i)$ in the first term is a polynomial of degree no greater than $k-1$ and thus can be expressed as a linear combination of $p_j(x)$, $j = 0,\cdots,k-1$. Therefore, again by the induction hypothesis, the first term is also 0. For $j = 0,1,\cdots,k-2$, then (15) is satisfied for any choice of α_{k+1} and β_k.

For $j = k-1$ the second term is still 0, and so we get the requirement that

$$\beta_k = \frac{\sum_{i=1}^{n} w_i\, x_i\, p_{k-1}(x_i) p_k(x_i)}{\sum_{i=1}^{n} w_i \left[p_{k-1}(x_i)\right]^2}$$

$$= \frac{\sum_{i=1}^{n} w_i \left[p_k(x_i)\right]^2}{\sum_{i=1}^{n} w_i \left[p_{k-1}(x_i)\right]^2}, \qquad (21)$$

the second form following from use of (18). For $j = k$ the third term vanishes, and we get,

$$\alpha_{k+1} = \frac{\sum_{i=1}^{n} w_i\, x_i \left[p_k(x_i)\right]^2}{\sum_{i=1}^{n} w_i \left[p_k(x_i)\right]^2}. \qquad (22)$$

Thus, we have the result that with β_k and α_{k+1} given by (21) and (22), the polynomial of degree $k+1$, $p_{k+1}(x)$, defined by (18), satisfies the orthogonality relation (15). This proves our assertion that a recurrence relation of the form (18) exists. We have assumed in the above that the denominators in (21) and (22) do not vanish. A denominator can vanish only if,

$$p_j(x_i) = 0,\ i = 1,\cdots,n, \qquad (23)$$

for some j. If (23) held for no j, we could generate an unending sequence of polynomials $p_j(x)$. Given n data points, we expect to be able to generate at most n independent polynomials $p_0(x),\cdots,p_{n-1}(x)$. Therefore, it is no surprise that we can show that if (18) is used to generate $p_n(x)$, then $p_n(x_i) = 0$, $i = 1,\cdots,n$.

Using (18), (21) and (22), we can generate the least-squares approximation

$$y_m(x) = \sum_{j=0}^{m} b_j\, p_j(x), \qquad (24)$$

with $$b_j = \frac{\omega_j}{\gamma_j}, \qquad (25)$$

where $$\omega_j = \sum_{i=1}^{n} w_i\, \bar{f}_i\, p_j(x_i),$$

and $$\gamma_j = \sum_{i=1}^{n} w_i \left[p_j(x_i)\right]^2.$$

In addition, it provides a convenient method of evaluating the approximation $y_m(x)$. It is not very hard to show that the recurrence

$$q_k(x) = b_k + (x - \alpha_{k+1})\, q_{k+1}(x) - \beta_{k+1}\, q_{k+2}(x),$$
$$k = m, m-1,\cdots,0,$$
$$q_{m+1}(x) = q_{m+2}(x) = 0, \qquad (26)$$

is such that $q_0(x) = y_m(x)$. Similar recurrences can be used to evaluate the derivative of $y_m(x)$.

When the data points are equally spaced and for the particular case $w(x) = 1$, the orthogonal polynomials are called Gram polynomials. In this case it is convenient to use an odd number of points $2L+1$

$$x_s = x_0 + sh,\ s = -L,\cdots,-1,0,1,\cdots,L.$$

It can be shown that $\alpha_{j+1} = 0$ for all j, so that the resulting recurrence relation is

$$\frac{1}{\varepsilon_{j+1}} p_{j+1}(s,2L) = \frac{s}{\varepsilon_j} p_j(s,2L) - \frac{\beta_j}{\varepsilon_{j-1}} p_{j-1}(s,2L),$$
$$j = 0,1,\cdots,$$
$$p_0(s,2L) = 1, \quad p_{-1}(s,2L) = 0,$$

where $$\beta_j = \frac{j^2\left[(2L+1)^2 - j^2\right]}{4(4j^2-1)},$$

$$\varepsilon_j = \frac{(2j)!}{(j!)^2}\frac{1}{(2L)^{(j)}}.$$

Before presenting an example of a least-squares orthogonal polynomial approximation in section 5, we develop and present here an algorithm for generating approximations of the form (24). Our technique is to use (18), (21) and (22) to generate the orthogonal polynomials or, more precisely, the values of these polynomials at the points x_i.

Input

$\{x_i, w_i, \bar{f}_i\}$ $\quad i = 1,\cdots,n$

m

Algorithm

for $i = 1,\cdots,n$ **do**

$p_0(x_i) \leftarrow 1;\ p_{-1}(x_i) \leftarrow 0;\ y_m(x_i) \leftarrow 0$ **endfor**

$\gamma_0 \leftarrow \sum_{i=1}^{n} w_i;\ \beta_0 \leftarrow 0$

for $j = 0,\cdots,m$ **do**

$\omega_j \leftarrow \sum_{i=1}^{n} w_i\, \bar{f}_i\, p_j(x_i)$

$b_j \leftarrow \omega_j / \gamma_j$

for $i = 1,\cdots,n$ **do**

$y_m(x_i) \leftarrow y_m(x_i) + b_j\, p_j(x_i)$ **endfor**

if $j = m$ **then stop**

$\alpha_{j+1} \leftarrow \sum_{i=1}^{n} w_i\, x_i \left[p_j(x_i)\right]^2 / \gamma_j$

for $i = 1,\cdots,n$ **do**

$p_{j+1}(x_i) \leftarrow (x_i - \alpha_{j+1})\, p_j(x_i) - \beta_j\, p_{j-1}(x_i)$ **endfor**

$$\gamma_{j+1} \leftarrow \sum_{i=1}^{n} w_i \left[p_j(x_i) \right]^2$$

$$\beta_{j+1} \leftarrow \gamma_{j+1} / \gamma_j$$

endfor
Output

$b_j, \ j = 0, \cdots, m$

$\alpha_j, \ j = 1, \cdots, m$

$\beta_j, \ j = 1, \cdots, m-1$

$y_m(x_i), \ i = 1, \cdots, n$ (smoothed values at the data points)

We note that α_j and β_j are listed as outputs in order to make it possible to use the recurrence relation (26) to compute $y_m(x)$ at points other than the given x_i.

5. An Example Of The Generation Of Least-Squares Approximations

Suppose we are given the empirical data

Table 1.

x_i	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\bar{f}_i$	5.1234	5.3057	5.5687	5.9378	6.4370	7.0978	7.9493	9.0253	10.3627

and we wish to find the best least-squares polynomial approximation to $f(x)$ with weight function $w(x) = 1$. Since the data points are equally spaced, we could use the Gram polynomials. Instead, however, we shall use the algorithm at the end of the section 4. Using this algorithm with $m = 5$ we calculate:

Table 2.

j	γ_j	ω_j	b_j	α_j	β_j
0	9	62.80767	6.97863		
1	0.6	3.80372	6.33953	$\frac{1}{2}$	$\frac{1}{15}$
2	$\frac{11.088}{360,000}$	2.05302	8.16435	$\frac{1}{2}$	$\frac{77}{1500}$
3	$\frac{32,076}{225 \times 10^5}$	0.00711	4.98754	$\frac{1}{2}$	$\frac{81}{1750}$
4	$\frac{45,045}{765,625 \times 10^3}$	0.00006	0.99883	$\frac{1}{2}$	$\frac{13}{315}$
5	$\frac{13}{6,250,000}$	0.0000003	0.12821	$\frac{1}{2}$	

and $y_5(0.1) = 5.1234$, $y_5(0.2) = 5.3056$

$y_5(0.3) = 5.5689$, $y_5(0.4) = 5.9374$

$y_5(0.5) = 6.4374$, $y_5(0.6) = 7.0976$

$y_5(0.7) = 7.9491$, $y_5(0.8) = 9.0255$

$y_5(0.9) = 10.3626$.

Using the results computed with the algorithm we then, as in (10), calculate

$$\delta_m^2 = \sum_{i=1}^{9} \left[\bar{f}_i - \sum_{j=0}^{m} b_j \, p_j(x) \right]^2 .$$

We obtain

$\delta_0^2 = 26.202$, $\delta_1^2 = 2.089$, $\delta_2^2 = 0.0355$

$\delta_3^2 = 0.000059$, $\delta_4^2 = 0.00000049$, $\delta_5^2 = 0.00000045$.

Then, using (12), we calculate

$\sigma_0^2 = 3.275$, $\sigma_1^2 = 0.298$, $\sigma_2^2 = 0.0059$

$\sigma_3^2 = 0.000012$, $\sigma_4^2 = 0.00000012$, $\sigma_5^2 = 0.00000015$

from which we conclude that $m = 4$ gives us the best least-squares approximation. This approximation is

$$y_4(x) = \sum_{j=0}^{4} b_j \, p_j(x) , \qquad (27)$$

with b_j's given by (25) and $p_j(x)$ given by (18). Although there is normally no computational need to do so, we can convert (27) into an approximation using powers of x, in which case we get

$$y_4(x) = 0.9988\,x^4 + 2.9898\,x^3 + 2.0172\,x^2$$
$$+0.9920\,x + 5.0010 . \qquad (28)$$

In fact the values given in the table at the start of the section are perturbations of the values of

$$f(x) = x^4 + 3x^3 + 2x^2 + x + 5 . \qquad (29)$$

The true values of $f(x)$ at the points given in the table are

Table 3.

x_i	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
f_i	5.1231	5.3056	5.5691	5.9376	6.4375	7.0976	7.9491	9.0256	10.3631

In order to show the effect of the ill-condition of the matrix G, let us now repeat the above computation using powers of x instead of orthogonal polynomials. Using (6) and (7), we wish to calculate $a_j^{(4)}$, $j = 0, \cdots, 4$. We get for G, the matrix of the g_{jk},

$$G = \begin{bmatrix} 9.0 & 4.5 & 2.85 & 2.025 & 1.5333 \\ 4.5 & 2.85 & 2.025 & 1.5333 & 1.20825 \\ 2.85 & 2.025 & 1.5333 & 1.20825 & 0.978405 \\ 2.025 & 1.5333 & 1.20825 & 0.978405 & 0.8080425 \\ 1.5333 & 1.20825 & 0.978405 & 0.8080425 & 0.67731333 \end{bmatrix}$$

$$= 9\begin{bmatrix} 1.00000 & 0.50000 & 0.31667 & 0.22500 & 0.17037 \\ 0.50000 & 0.31667 & 0.22500 & 0.17037 & 0.13425 \\ 0.31667 & 0.22500 & 0.17037 & 0.13425 & 0.10871 \\ 0.22500 & 0.17037 & 0.13425 & 0.10871 & 0.08978 \\ 0.17037 & 0.13425 & 0.10871 & 0.08978 & 0.07526 \end{bmatrix}. \quad (30)$$

From (30) we see that $\frac{1}{9}G$ is quite close to (9), as we would expect since the points x_i are all in the interval $(0,1)$ and are equally spaced. For the determinant of G we get

$$\det G = 0.000000014.$$

Therefore, we expect that the errors incurred in the calculation of G will cause substantial errors in the solution of the normal equations. We emphasize that the orthogonal-polynomial and powers-of-x formulations are two ways of stating precisely the same problem. Therefore, any difference between (28) and the solution of (7) will be entirely due to different computational techniques.

When (6) is used, the right-hand side of (7) is

$$\rho_0 = 62.8077 \qquad \rho_1 = 35.20757 \qquad \rho_2 = 23.944287$$
$$\rho_3 = 17.8176647 \quad \rho_4 = 13.93266027.$$

If we solve the normal equations using Gaussian elimination and carry six decimal places throughout the computation, we get, rounding the results to four decimal places,

$$a_4^{(4)} = 0.9672 \qquad a_3^{(4)} = 3.0522 \qquad a_2^{(4)} = 1.9763$$
$$a_1^{(4)} = 1.0020 \qquad a_0^{(4)} = 5.0003, \qquad (31)$$

which in the case of $a_4^{(4)}$ and $a_3^{(4)}$ have errors for larger than those in (28).

It is important that we clearly distinguish the two types of errors considered here. On the one hand, the ill condition of G causes the difference between the calculated coefficients given by (28) and (31). On the other hand, the difference between the coefficients in (28) and the true coefficients (29) is due to the inherent empirical errors in the data.

6. Conclusions

We have presented some material on least-squares approximations. Approximating continuous functions by least-squares techniques is, of course, of great theoretical interest. We shall also see orthogonal polynomials play an important role in discrete least-squares approximations. Then, we develop an algorithm for generating approximations. The best least-squares polynomial approximation to the given function is obtained using the orthogonal polynomials.

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Applying the Queue Theory in Bank Service Centers

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ABSTRACT: *Nowadays, all of the places like shopping center, gasoline or at banks, there are more than one line in service center. In this paper, the queue theory is applied to reduce the waiting time of customers at the bank. Single queue and single service center (M/M/1) queuing model and single queue and multiple service center (M/M/s) queuing model are analyzed. By using these two models are calculated their usefulness in banks. This paper reviews the two types of queuing models for single and multiple servers. We can compare this average with that of queuing model. Two different models are used to estimate a queue length: a single-queue and single-servicer model, single-queue and multiple-server model. It compares the difference between multiple services and single service. In the KBZ bank, one remittance server and one queue, two remittance servers and one queue are considered as the case study. In this case study, using a queue model at a bank describes the difference between single service center and multiple service centers. This paper demonstrate how well queuing models can be used to analyze and understand the behavior of the queues particular in the banking industry.*

Keywords: Queuing System; FCFS; Multiple Servers; Poisson arrival.

1. INTRODUCTION

Today, everyone in life has to deal with queuing problems. This problem is found in many areas. The queuing theorem is about the importance of society. By using it, we will see that it saves time and becomes more organized. Moreover, use the queuing theory to reduce customer waiting time and improve the better system. The queue model provides ideas for ways to increase customers satisfaction.

In the bank, some people want to jump from the current queue to the shortest one. So, this behavior is not actually saving their time. We studied the queue theory at the bank. To solve this problem, we introduce single queue multiple service model for the bank. We collect the data of the existing system for one week at KBZ bank at Magway Division.

This paper will illustrate using the quantitative mathematical approach to explain the queuing scenarios. The bank has its headquarters and employs over 100 staff throughout.

However, queuing problems are inevitable and KBZ is no exception. While KBZ maintains the largest market share in this part of the region, queuing or the long waiting lines during business hours is ab unbearable sight. We all may have experienced what is like to be standing in the queue. For the sake of our analysis, we have chosen to take a study on one of the banking services centers of KBZ in Magway division at the heart of the Capital Business District area of Magway city also known as the industrial hub of Magway.

Queuing is evident when there is inefficiency in the service delivery mechanism, or lack of business philosophy of customer centric. Fundamentally, queuing is obvious when the rate of customers arriving is greater than the time it takes a customer to be served.

KBZ has embarked on improving banking services by harnessing the power of technology with the recent introduction of online and mobile banking services to ease the burden of long queues. We have constructed a decision tree in Figure to help us assess and evaluate considering best practice the option that maximize value to the customer and minimizes opportunity productivity loss created by the long queues.

2. METHODOLOGY

We expressed the formula of M/M/1 model and M/M/S model. And also, the comparison of these two models.

2.1. The Formula for Queuing Model M/M/1

The M/M/1 queuing model is a queuing model where the arrivals follow a Poisson process, service time are exponentially distributed and there is one server. The time taken to complete a single service is exponentially distributed with parameter μ.

The customers arrive randomly over time and wait in a queue(line), and upon beginning service, each customer spend a random amount of time in service before departing. The following formula is M/M/1 model.

The unit of bank may consist single server in cash counter, enquiry, and issues of Cheque etc. These units have and single waiting line of customers (M/M/1 queuing model). For such a model the following assumptions are made:

(i) Inter arrival of the customers during time period $[t, t + \Delta t)$ only depends on the length of the time period 's'. Where the arrival rate of customers and service rate of server are constants regardless of the state of the system (busy or idle).
(ii) Service times are exponentially distributed.
(iii) Customers are served FIFO (First in First Out) basis. No customer leaves the queue without being served. All customers arriving in the queuing system will be served approximately equally distributed service

time, whereas customer choose a queue randomly, or choose or switch to shortest length queue.

In Queuing Model M/M/1 for Single Service Center,

Let we define $\lambda =$ arrival rate and $\mu =$ service rate.

The formula for utilization factor, $\rho = \frac{\lambda}{\mu}$ and

The Probability of zero customer in queue is $P_0 = 1 - \rho$

Probability of having 'n' customers in queue is $P_n = P_0 \rho^n$

Average number of customers in the systems is $L_s = \frac{\rho}{1-\rho}$

Average number of customers in the queue is

$$L_q = \frac{\rho^2}{1-\rho}$$

Average number of waiting time in the queue is $W_q = \frac{L_q}{\lambda}$

Average time spent in the system is $w_s = \frac{L_s}{\lambda}$ respectively.

And then, we show the model M/M/1 for single service center in the following figure-1.

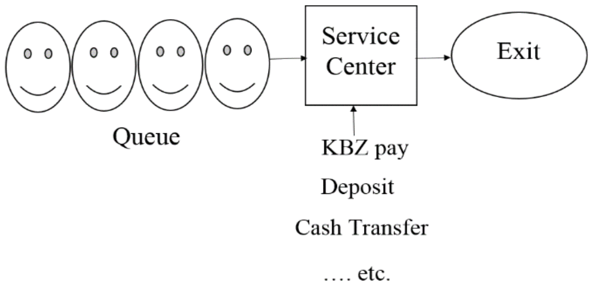

Figure 1. Single queue and single service center

2.2. The Formula for Queueing Model M/M/s

In queue theory, a discipline within the mathematical theory of probability, the M/M/s queue is a multiple server queue model. Queuing theory to analysis of waiting line in healthcare setting. Most of healthcare system have excess capacity to accommodate random variations, so queuing analysis can be used as short terms measure, or for facility and resource planning. The following formula is M/M/1 model.

The bank may consist of multiple units to give service to customers. Some of the units like loan sanction, identity verification, etc., have multiple servers with single queue or multiple waiting lines of customers (M/M/s queuing model). For such a model the following assumptions are made:

(i) Inter arrival of the customers during time period $[t, t + \Delta t)$ only depends on the length of the time period 's'. Where the arrival rate of customers and service rate of server are constants regardless of the state of the system (busy or idle).
(ii) Service times are exponentially distributed.
(iii) Customers are served FIFO (First In First Out) basis. No customer leaves the queue without being served. All customers arriving in the queuing system will be served approximately equally distributed service time.

In Queueing Model M/M/s for two Service Center, we suppose as

$\lambda =$ arrival rate, $\mu =$ service rate,

$s = number\ of\ servers$ and service rate for two servers = $s\mu$

And then, Overall system utilization factor is $\rho = \frac{\lambda}{s\mu}$,

$Let\ \gamma = \frac{\lambda}{\mu}$

The Probability of zero customer in queue is

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{(s\rho)^n}{n!} + \frac{(s\rho)^s}{s!\,(1-\rho)} \right]^{-1}$$

Probability of having 'n' customers in the system is $P_n = P_0 \rho^n$

Average number of customers in the queue is

$$L_q = P_s \frac{\rho}{(1-\rho)^2}$$

Average number of customers in the system is

$$L_s = L_q + \frac{\lambda}{\mu}$$

Average number of waiting time in the queue is $W_q = \frac{L_q}{\lambda}$

Average time spent in the system is $w_s = \frac{L_s}{\lambda}$

So, we shown the model MMs for two service centers in the following figure-2.

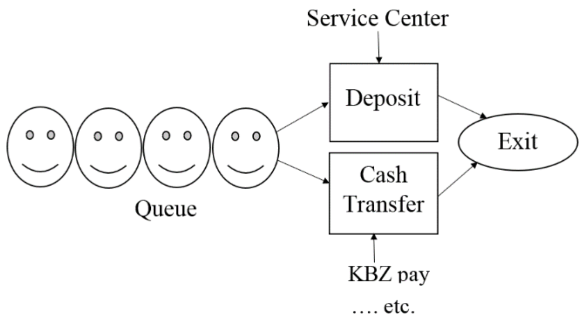

Figure 2. Single queue and multiple service centers

3. Experiment Results For Bank Service Problem

In KBZ bank at Magway Division, the results of one week show the following results for the first step. The first table is the application of M/M/1 model and the second table is the application of M/M/S model. Customers arrival is a Poisson distribution that provides customers with services based on First Come First Service (FCFS). In addition, the system has no priority customers. Servers are serviced continuously and are not affected by external forces. In this paper, we calculate the problems by using M/M/1 for single service center and using M/M/s for two service centers. The result as shown in the tables and figures also.

3.1. Queuing Model M/M/1 for Single Service Center

From the article 2.1,

$$\rho = \frac{\lambda}{\mu} = \frac{30}{36} = 0.8333$$

The Probability of zero customer in queue

$$P_0 = 1 - \rho = 1 - 0.8333 = 0.1667$$

Average number of customers in the systems

$$L_s = \frac{\rho}{1-\rho} = \frac{0.8333}{1-0.8333} = 4.9988\ customer$$

Average number of customers in the queue

$$L_q = \frac{\rho^2}{1-\rho} = \frac{(0.8333)^2}{1-0.8333} = 4.1655\ customer$$

Average number of waiting time in the queue

$$W_q = \frac{L_q}{\lambda} = \frac{4.1655}{30} = 0.1389\ hr = 8.4334\ min$$

Average time spent in the system

$$w_s = \frac{L_s}{\lambda} = \frac{4.9988}{30} = 0.1666\ hr = 9.996\ min$$

According the calculation as reported in Table (1) the finding of the result show that the optimum configuration is realized when the number of services reach two at time 9:00 am to 11:00 am. With this arrangement, the average proportion of time that server is found to be 0.1667. Moreover, the average number of customers' waiting in the queue (L_q) and waiting in the system (L_s) are found to be 4.1655 and 4.9988 customers respectively. And then, Average number of waiting time in the queue (W_q) is 8.3340 and Average time spent in the system (W_s) is 9.9960.

Similarly, the other two time 11:00 am to 1:pm and 1:00 pm to 3:00 pm.

Table 1. The obtained results by using queuing model M/M/1

Time	Arrival rate	P_0	L_q	L_s	W_q	W_s
9:00 am to 11:00 am	30	0.1667	4.1655	4.9988	8.3340	9.9960
11:00 am to 1:00 pm	25	0.1935	3.3615	4.1680	8.0676	10.0032
1:00 am to 3:00 pm	20	0.2308	2.5636	3.3328	7.6908	9.9984

We construct the figure (3) from the above table (1).

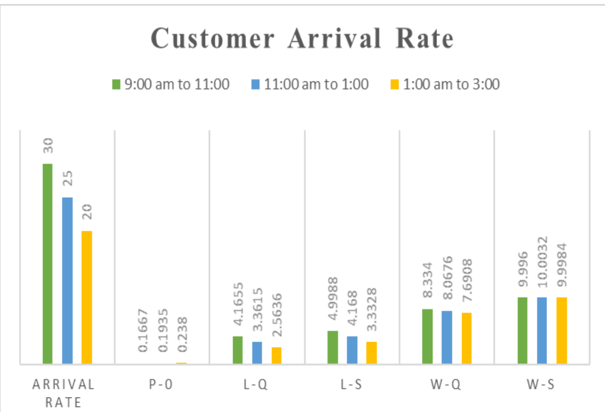

Figure 3. Customer arrival rate by using queuing model M/M/1

Results of Mean for M/M/1

Means for probability of zero customer in queue is 0.1970.

Mean for average number of customers the in system is 4.1665.

Mean for average number of customers in the queue is 3.3635.

Mean for average number of waiting time in the queue is 8.0308.

Mean for average time spent in the system is 9.9992.

We express the results of Mean for M/M/1 in the following table (2).

Table 2. Results of Mean for M/M/1

Results of Mean	M/M/1
P_0	0.1970
L_s	4.1665
L_q	3.3635
W_s	8.0308
W_q	9.9992

3.2. Queuing Model M/M/s for two Service Center

λ = arrival rate = 60 customers per hour for two service Center (i.e., 30 customers in service)

μ = service rate = 36 customer per server per hour

$s = number\ of\ servers = 2$

Service rate for two servers $= s\mu = 72$ customers

$$\rho = \frac{\lambda}{s\mu} = \frac{60}{72} = 0.8333$$

$$Let\ \gamma = Utilization\ factor\ for\ one\ server$$

$$\gamma = \frac{\lambda}{\mu} = \frac{60}{36} = 1.6667$$

The Probability of zero customer in queue

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{(s\rho)^n}{n!} + \frac{(s\rho)^s}{s!\,(1-\rho)}\right]^{-1} = 0.0909$$

Probability of having 'n' customers in the system,

$$P_n = P_0 \rho^n$$

Average number of customers in the queue

$$L_q = P_s \frac{\rho}{(1-\rho)^2} = (0.0631)\frac{0.8333}{(1-0.8333)^2} = 1.8923\ customer$$

Average number of customers in the system

$$L_s = L_q + \frac{\lambda}{\mu} = 1.8923 + 1.6667 = 3.559\ customer$$

Average number of waiting time in the queue

$$W_q = \frac{L_q}{\lambda} = \frac{1.8923}{60} = 0.0315\ hr = 1.8900\ min$$

Average time spent in the system

$$w_s = \frac{L_s}{\lambda} = \frac{3.559}{60} = 0.0593\ hr = 3.558\ min$$

According the calculation as reported in Table (3) the finding of the result show that the optimum configuration is realized when the number of services reach two at time 9:00 am to 11:00 am. With this arrangement, the average proportion of time that server is found to be 0.0909. Moreover, the average number of customers' waiting in the queue (L_q) and waiting in the system (L_s) are found to be 1.8923 and 3.559 customers respectively. And then, Average number of waiting time in the queue (W_q) is 1.8900 and Average time spent in the system (W_s) is 3.558.

Similarly, the other two time 11:00 am to 1:pm and 1:00 pm to 3:00 pm.

Table 3. The obtained results by using queuing model M/M/s

Time	Arrival rate	P_0	L_q	L_s	W_q	W_s
9:00 am to 11:00 am	60	0.0909	1.8923	3.559	1.8900	3.558
11:00 am to 1:00 pm	50	0.0940	1.3161	2.9290	1.5793	3.5148
1:00 am to 3:00 pm	40	0.1130	0.9660	2.5045	1.4490	3.7568

We construct the figure (4) from the above table (3).

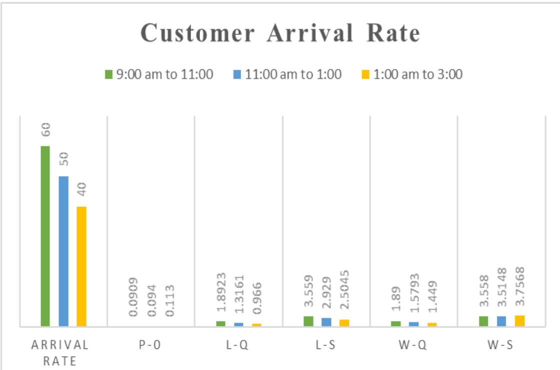

Figure 4. Customer arrival rate by using queuing model M/M/s

Due to the high arrival rate between 9:00 am and 11:00 am, the service rate is reduced and the waiting time is increased. The arrival rate decreases between 1:00 pm and 3:00 pm hours, increasing the service rate and decreasing the waiting time.

Result of Mean for M/M/s

Mean for probability of zero customers in the system is 0.0993,

Mean for average number of customers in the system is 2.9975.

Mean for average number of customers in the queue is 1.3915.

Mean for average number of waiting time in the system is 1.6394.

Mean for average time spent in the system is 3.6099.

We express the results of Mean for M/M/s in the following table (4).

Table 4. Results of Mean for M/M/s

Results of Mean	M/M/s
P_0	0.0993
L_s	2.9975
L_q	1.3915
W_s	1.6394
W_q	3.6099

4. Comparison Of Results With Tables And Figures

4.1. Experiment of Single Service and Multiple Services

After the experiment, we can find out the results for two models. This shown in the following tables and figures.

Table 5. Experiment result for two model

Results of Mean	M/M/1	M/M/s
P_0	0.1970	0.0993
L_s	4.1665	2.9975
L_q	3.3635	1.3915
W_q	8.0308	1.6394
W_s	9.9992	3.6099

Comparing the M/M/1 model with the M/M/s model not only reduces the waiting time in the system and queue, but also reduces mean probability of number of customers. If you upgrade from one server to two servers, it will be even lower if you increase it a lot, such as three or four servers etc.

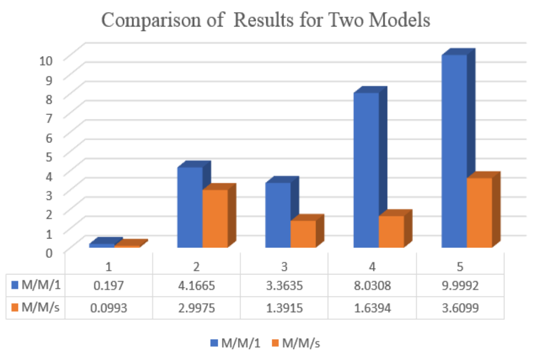

Figure 5. Comparison of result for two models

5. Conclusions

In this paper, we calculate the bank service center at KBZ bank. The queuing system is very useful throughout society. Based on the above results, we can see that the waiting time is reduced in the system. We use queuing theory to extend the behavior of service centers for banks. This queuing model helps customers prepare for the best decision on the service organization and structures. The more service centers there are, the less waiting time for customers. By this practical analysis of this research, we suggest that the bank managers can increase the number of servers. In this way, this research can contribute to the betterment of a bank in terms of its functioning.

The research tries to develop a practical queue model in line with the working arrangements of the bank. The findings of the research show that the arrival rate of customers follows a Poisson probability distribution and the service rate of the servers follows an exponential probability distribution. Moreover, the simulation run shows that the total cost based on waiting and based on the systems are found to be optimum with more servers. The total customers waiting in the systems or in the bank also relatively low as compared with the total number of customers waiting in the systems when the numbers of servers are two. Based on the research finding it is recommended that the bank should be use more servers so that it can operate with optimum cost. But it needs to improve the utilization of the servers utilization.

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Optimizing Integer Programming Problem Using Branch and Bound Method

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ABSTRACT: *In this paper, the profit of a company was calculated by using one of integer programming method. The integer programming model is developed for profit optimization. One of the integer programming techniques, the Branch and Bound method was used. In solving integer programming problem based on a given problem by using the branch and bound method, it was applied with the graphical method and calculated this method by using the Excel. And then, the result was drawn with the tree diagram. Therefore, the profit of the company was shown the maximum profit in a week. The result from the graphic calculation was checked by using the LINGO software. LINGO can solve integer optimization models faster, easier and more efficient.*

Keywords: Integer programming; Graphical method; Excel; Branch and Bound method; LINGO software

1. INTRODUCTION

An integer programming problem is a mathematical optimization or feasibility program that is limited to some or all variables as numbers. Integer programming problems often involve linear limit function optimization and linear constraints; non-negativity conditions and all or some variables need to be integers. The most common way to solve integer programming problems is the branch and bound method [3].

A search procedure is the Branch and Bound method. It is a systematic way of sorting out the numbers of a possible solution. Branch and Bound method solves the problems that can be presented as the leaves of a search tree. Each node is a possible solution to a problem. This method solved the graph with excel to obtain the maximize optimal value of the integer programming problem [1] [2].

It is ideal for any computer program because it requires a lot of calculations to solve a program. We used the computer program called LINGO software. LINGO is the mathematical modeling language desigened particularly for formulationg and solving a wide variety of optimization problem [2]. LINGO software is designed to effectively build and solve linear, nonlinear and integer optimization models. In this paper, the integer programming problem was solved with LINGO [6].

2. INTEGER PROGRAMMING

An extension of linear programming is integer linear programming. Integer-programming models appear in every relevant field of mathematical programming [4].

3. METHODOLOGY AND DESIGN

This paper is wanted the profit of the company in a week and its model was integer programming model. So, it is solved by the graphical method with Excel to get the maximize profit of this problem, and showed the tree diagram to complete the solutions. Then, Lingo software is used for rechecking the result [6].

3.1. Integer Programming Model

An integer programming model is a linear programming problem that requires some or all variables to be non-negative integers. It is to take an integer value in one or more decision variables in the final solution [5].

3.2. Basic Component of the Integer Programming Problem

The integer programming required the following various components to make integer programming model,

(i) Objective function and
(ii) Constraints are linear
(iii) All decision variables are non -negative
(iv) Some or all decision variables are integers

3.3. General Form of the Integer Programming

(i)

$$Optimize \qquad \sum_{i=1}^{n} a_i x_i$$

(ii)

$$Subject\ to \qquad \sum_{i=1}^{n} b_{ji} x_i \le c_j$$

(iii) $x_i \ge 0\ , i = 1,2, \ldots, n\ , (j = 1,2, \ldots, m)$ and

(iv) x_i $are\ integer\ values$

The level of activity $x_i(\ i = 1,2, \ldots, n\ )$ are decision variables.

a_i= incease in the objective function that would result from each unit enhace in level of activity *i*.

b_{ji} =amount of resource *j* consumed by each unit of activity *i*

c_j = amount of resource j that is available for allocation to activities (right hand side of the equation)

a_i, b_{ji} and $c_j$are the input constants for the model and referred to as the parameters of the model.

$a_i = (a_1, a_2, \ldots, a_n)$ is a row vector,

$x_i = (x_1, x_2, \ldots, x_n)$ is a column vector,

b_{ji} is a matrix of order $m \times n$ and

$c_j = (c_1, c_2, \ldots, c_m)$ is a column vector.

3.4. Implementation of Branch and Bound method

The Branch and Bound technique include separating the feasible solution into smaller parts unti the objection function in the given problem.

Branch- If the value of x_i are one or more fractional values in the optimal solution, the problem is divided into two new sub-problems based on the value of x_i.

k_i=the fractional value of x_i

$$k_i \leq \lfloor k_i \rfloor$$

$$\text{and } k_i \geq \lfloor k_i \rfloor + 1$$

Sub-problem 1

$$MAX\ Z = \sum_{i=1}^{n} a_i x_i$$

$$Subject\ to \quad \sum_{i=1}^{n} b_{ji} x_i \leq c_j$$

$$k_i \leq \lfloor k_i \rfloor$$

and $x_i \geq 0. x_i$ are integer.

Sub-problem 2

$$MAX\ Z = \sum_{i=1}^{n} a_i x_i$$

$$Subject\ to \quad \sum_{i=1}^{n} b_{ji} x_i \leq c_j$$

$$k_i \geq \lfloor k_i \rfloor + 1$$

and $x_i \geq 0. x_i$ are integer.

Bound - The optimal value of the objective function of sub-problem 1 and sub-problem 2. The lower bound is the rounded off solution value of the objective function and the upper bound is the corresponding to the integer programming problem solution.

3.4.1. Procedure of the Branch and Bound method

Step 1 - The problem is solved by using the graph with excel to obtain an optimal solution for the given problem.

Step 2 - If one or more decision variables are not integer values in optimal solution, then divide the problem into two parts (sub-problems).

Step 3 - Solve the two sub-problems to obtain the optimal solution.

Step 4 - If the decision variable values in two sub-problems are integers, the highest value of the objective function in these two problems is the optimal solution.

3.4.2. Flowchart of the Problem

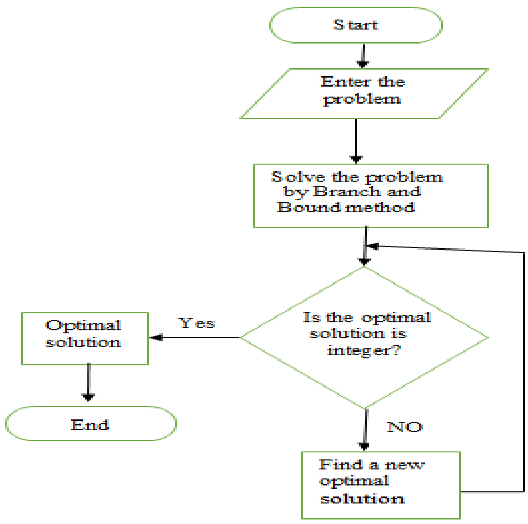


Figure 1. System flowchart for branch and bound method

4. Finding And Research

We solve an integer programming problem by using Branch and Bound method with graphically and LINGO software.

4.1. Solving the Integer Programming Problem

The company produced two types of bags, handbag and backpack. The raw materials required for each batch of handbags are twice as that of backpacks. The raw material is available only 60 units per week. Each batch of handbags requires 3 machine hours and 2 labour hours and each batch of backpacks requires 2 machine hours and 3 labour hours. The company has used mostly 34 machine hours and 33 labour hours that are available in a week. The profit is $3 each batch of handbags and $4 each batch of backpacks. How many batches of each type of bags should the company produce in a week to maximize profits?

(i) Define decision variable

$Let\ x_1$ = Number of batches of handbags produced per week

x_2 = Number of batches of backpacks produced per week

Z = the total profit of the problem

If the profit is $3 each batch of handbags and $4 each batch of backpacks, then the objective function can be written as;

$$Maximize \quad Z = 3x_1 + 4x_2$$

(ii) The next step is to identify the constraints.

The constraints relate to raw materials, machine hours and labour hours.

Raw materials require for each batch of handbags are twice as that of backpacks, then the constraint can be started;

$$x_1 + 2x_2 \leq 60$$

Each batch of handbags requires 3 machine hours and each batch of backpacks requires 2 machine hours and the total machine hours available in a week are 34, then the constraint can be written as;

$$3x_1 + 2x_2 \leq 34$$

Each batch of handbags requires 2 labour hours and each batch of backpacks requires 3 labour hours and the total labour hours available in a week are 33, then the constraint can be written as;

$$2x_1 + 3x_2 \leq 33$$

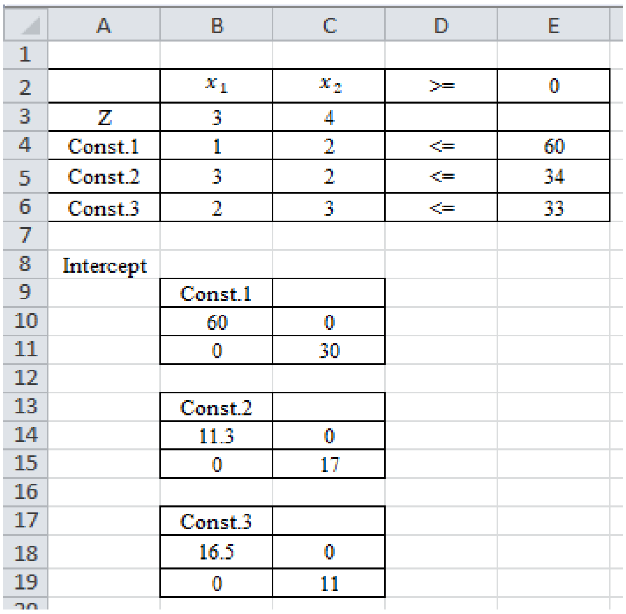
We have the non-negativity variables $x_1 \geq 0$, $x_2 \geq 0$, x_1, x_2 are integers

(iii) Calculate the integer linear programming model

$$\begin{aligned} &Maximize \quad Z = 3x_1 + 4x_2 \\ &Subject\ to \\ &\quad x_1 + 2x_2 \leq 60 \\ &\quad 3x_1 + 2x_2 \leq 34 \\ &\quad 2x_1 + 3x_2 \leq 33 \end{aligned}$$

$and\ x_1 \geq 0, x_2 \geq 0$, x_1, x_2 are integers

(iv)Solve the problem with graphically using Excel
Const.1 is represented constraint 1.
Const.2 is represented constraint 2.
Const.3 is represented constraint 3.



	A	B	C	D	E
1					
2		x_1	x_2	>=	0
3	Z	3	4		
4	Const.1	1	2	<=	60
5	Const.2	3	2	<=	34
6	Const.3	2	3	<=	33
7					
8	Intercept				
9		Const.1			
10		60	0		
11		0	30		
12					
13		Const.2			
14		11.3	0		
15		0	17		
16					
17		Const.3			
18		16.5	0		
19		0	11		

Figure 2. Entering data in the spreadsheet

Initially, to find x_1 so x_2=0 and
to find x_2, x_1= 0 in Const. 1
Select the Const.1, choose "Insert menu" and click "Scatters with Straight lines"
Right click on the picture and choose "Select Data"
click "add" and the box is shown in Fig.

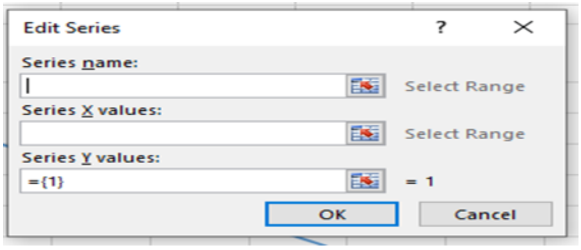


Figure 3. Edit Series

For Const.2, select the *x* values and into "Series *x* values" select the *y* values and into "Series *y* values" and click "OK" button and Const.3 use the same way as Const.2.

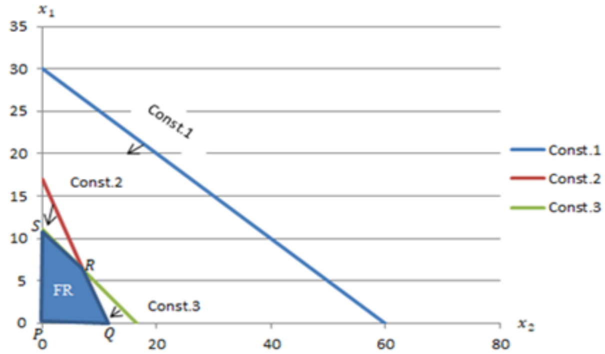


Figure 4. Graphic solution for the problem

This shaded area is called the feasible region, it is satisfied all the constraints. The solution space is the feasible region.
Point Q exist on the x_1 axis so x_2=0
Point S exist on the x_2 axis so x_1=0
Point R =

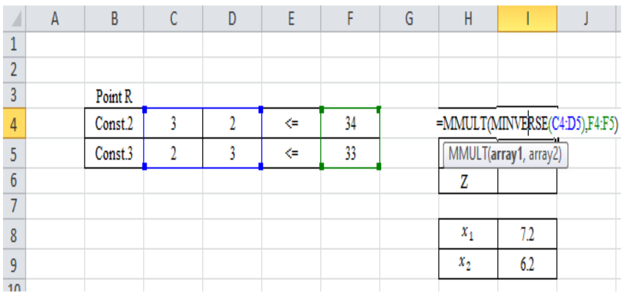


Figure 5. The value of the points

To obtain the value of Z, substitude the value of x_1 and x_2 in the objective function.

$$Z = 3x_1 + 4x_2$$
$$Z = 3*7.2+4*6.2$$

Table 1. The solution of the problem

	x_1	x_2	Z
Point P	0	0	0
Point Q	11.3	0	33.9
Point S	0	11	44
Point R	7.2	6.2	46.4

In points P, Q, R and S, point R is the highest value of the objective function (Z). So, the optimal solution is

$$x_1 = 7.2, x_2 = 6.2 \; and \; Z = 46.4$$

If the optimal solution obtained the decision variables are not integer, then divide into the problem two parts (sub-problems).

In the problem, the decision variables x_1 *and* x_2 are not integer so choose one decision variable $x_1 = 7.2$

In this case, the two integers very close to 7.2 would be 7 and 8, so $x_1 \leq 7$ *and* $x_1 \geq 8$.

Sub-problem 1;

$$Maximize \quad Z = 3x_1 + 4x_2$$
$$Subject\ to$$
$$x_1 + 2x_2 \leq 60$$
$$3x_1 + 2x_2 \leq 34$$
$$2x_1 + 3x_2 \leq 33$$
$$x_1 \leq 7$$

Sub-problem 2;

$$Maximize \quad Z = 3x_1 + 4x_2$$
$$Subject\ to$$
$$x_1 + 2x_2 \leq 60$$
$$3x_1 + 2x_2 \leq 34$$
$$2x_1 + 3x_2 \leq 33$$
$$x_1 \geq 8$$

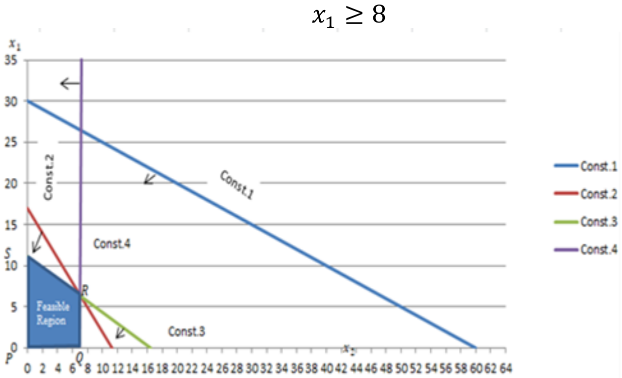


Figure 6. Graphic solution for sub-problem 1

To find the optimal solution of the subproblem 1 use the same way above the problem.

For sub-problem 1; $x_1 = 7, \; x_2 = 6.3 \; and$

$Z_1 = 46.3$ (use one decimal place)

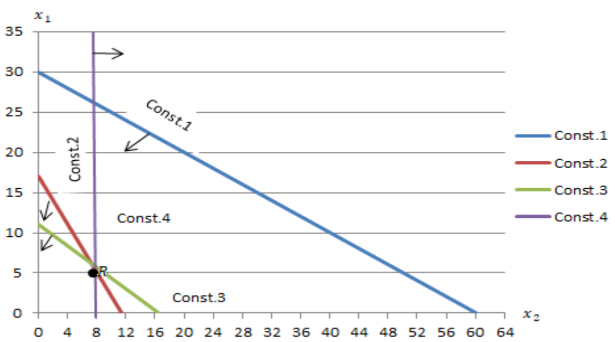


Figure 7. Graphic solution for sub-problem 2

For sub-problem 2; $x_1 = 8, \; x_2 = 5 \; and$

$Z_2 = 44$

This step is known as branching.

In sub-problem 1, the decision variable $x_2 = 6.3$ is not integer, so branch into sub-problem 1 say 1(a) and 1(b) for $x_2 \leq 6$ and $x_2 \geq 7$.

Sub-problem1(a)

$$Maximize \quad Z = 3x_1 + 4x_2$$
$$Subject\ to$$
$$x_1 + 2x_2 \leq 60$$
$$3x_1 + 2x_2 \leq 34$$
$$2x_1 + 3x_2 \leq 33$$
$$x_1 \leq 7$$
$$x_2 \leq 6$$

Sub-problem1(b)

$$Maximize \quad Z = 3x_1 + 4x_2$$
$$Subject\ to$$
$$x_1 + 2x_2 \leq 60$$
$$3x_1 + 2x_2 \leq 34$$
$$2x_1 + 3x_2 \leq 33$$
$$x_1 \leq 7$$
$$x_2 \geq 7$$

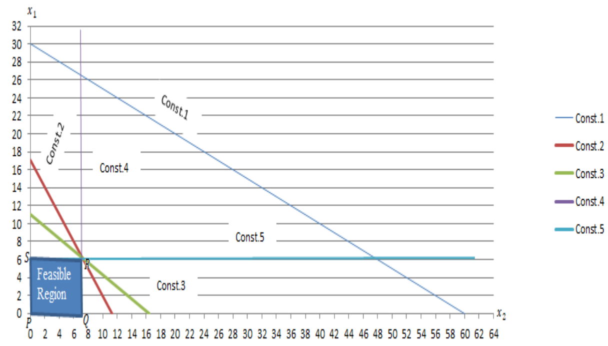


Figure 8. Graphic solution for sub-problem 1(a)

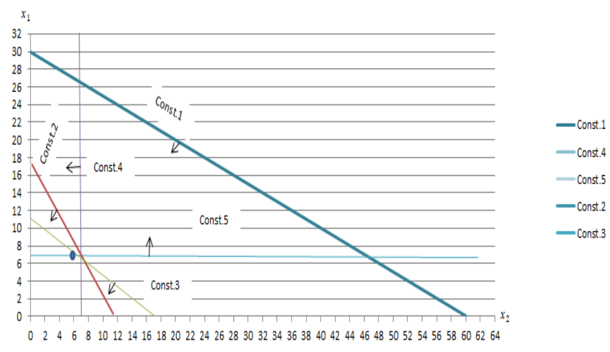


Figure 9. Graphic solution for sub-problem 1(b)

For sub-problem 1(a); $x_1 = 7, \; x_2 = 6 \; and$

$Z_3 = 45$

For sub-problem 1(b); $x_1 = 6, \; x_2 = 7 \; and$

$Z_4 = 46$

At the end of the search, the objective function value (in the maximization problem) is chosen as the highest integer value, the optimal solution is $x_1 = 6$, $x_2 = 7 and \; Z_4 = 46$.

If the company produces 6 batches of handbags and 7 batches of backpacks per week, it obtains the maximum profit is $46.

4.2. The Tree Diagram of Branch and Bound

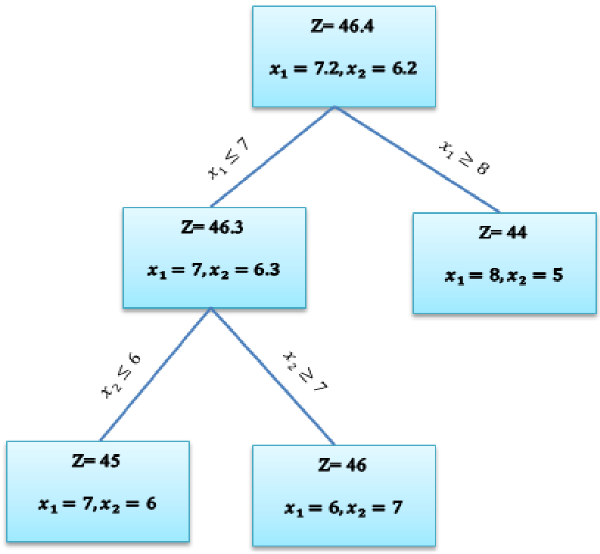


Figure 10. Complete the branch and bound solution

The branch and bound method find a minimum path to reach the optimal solution for a given problem. It doesn't repeat nodes while exploring the tree. The time complexity of the branch and bound mehtod is less when compared with other method.

4.3. Checking the Result by Using Lingo

Open the LINGO and input the data of the problem; X1and X2 are represented of the decision variables x_1 *and* x_2 .The first line is the objective function (without including Z variable) and indicates that it is to be maximized and the next three lines are functional constraint lines.

LINGO requires to use the asterisk for the multiplication and at the end of the lines we add a semi-colon.

The values of X1 and X2 are fraction, so use @GIN (any positive integer)

Then, use the "Solve" command from the "Solver menu", click on the button or press "Ctrl+U".

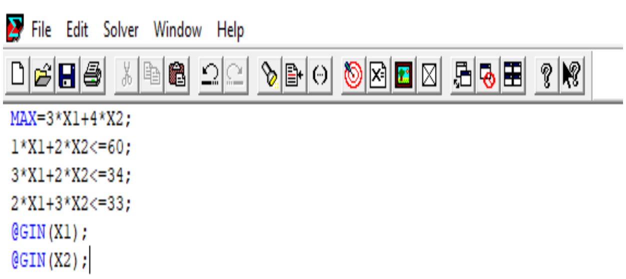


Figure 11. LINGO input file for the problem

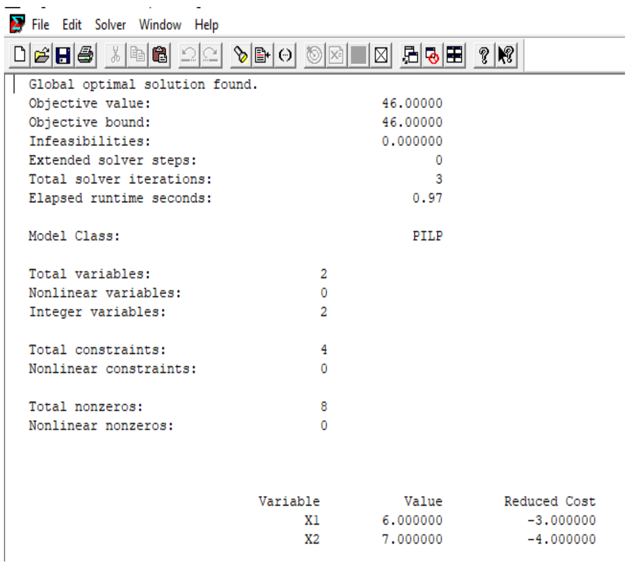


Figure 12. The optimal solution for the problem

This window displays the values of each variable and it will produce the optimal value of the objective function.

The optimal solution is $x_1 = 6,\ x_2 = 7\ and\ Z = 46$ **.**

5. Conclusions

When we solved the problem by using the Branch and Bound method, the optimal solutions may be integers or not integers. If the optimal solutions are not integers, the problem is divided into two parts to obtain the optimal solution and then the problem is calculated step by step to get the decision variables are integers. When the values of the decision variables are integers, we received the optimal solutions of the problem. It can decide how much the batches of handbacks and backpacks produced in a week to obtain the maximize profit. So, the branch and bound method is one of the most common technique to solve integer programming problem. This problem was solved by using the LINGO software. The optimal values using the branch and bound method and LINGO are the same but LINGO can cut down the time. It can formulate the integer programming problems quickly. Therefore, LINGO software can give the benefit of the exact values for solving the integer programming problems. The knapsack-capital budgeting problem, warehouse location problem, scheduling, sequencing and several other problems can also be solved as linear integer optimization problems.

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